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Question Paper Code : 82376

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2025.

Third Semester

Electronics and Communication Engineering

MA 3355 — RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to: Electronics and Telecommunication Engineering)

(Regulations 2021)

(Normal Table is to be permitted)

Time : Three hours

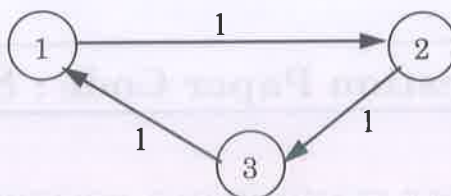
Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. An integrated circuit factory has three machines X, Y and Z . Test one integrated circuit produced by each machine. Either a circuit is acceptable (a) or it fails (f). An observation is a sequence of three test results corresponding to the circuits from machines X, Y and Z , respectively. What are the elements of the sample space of this experiment?
2. Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is a zero with probability 0.8, independent of any other bit. What is the probability that a code word contains exactly three ones?
3. The joint probability mass function of two random variables X and Y is given by $P_{XY}(x,y) = \begin{cases} k(2x+y), & x=1,2 \text{ and } y=1,2 \\ 0, & \text{otherwise} \end{cases}$ where k is a constant. What is the value of k ?
4. Prove that the $\text{cov}(X,Y) = 0$ if X and Y are independent random variables.
5. Cars, trucks and buses arrive at a toll booth as independent Poisson processes with rates $\lambda_c = 1.2$ cars/minute, $\lambda_t = 0.9$ trucks/minute and $\lambda_b = 0.7$ buses/minute. In a 10-minute interval, what is the PMF of N , the number of vehicles (cars, trucks, or buses) that arrive?

6. Identify the transient states and recurrent states whose state-transition diagram is shown in the following figure.



7. Prove that any two intersection of subspaces of vector space V is a subspace of V .
8. Is it possible to write the vector $(-2, 0, 3)$ as a linear combination of the two vectors $(1, 3, 0)$ and $(2, 4, -1)$? Justify.
9. Is the function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (x + 2y, 3x - y)$ where $(x, y) \in \mathbb{R}^2$ a linear operator?
10. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the orthogonal projection onto the XY -plane. Identify the kernel and range of T .

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Suppose traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.8 a driver will find the second light to have the same colour as the first. Assuming the first light is equally likely to be red or green, what is the probability $P(G_2)$ that the second light is green? Also, what is $P(W)$, the probability that you wait for at least one light? Lastly, what is $P(G_1/R_2)$, the conditional probability of a green first light given a red second light? (8)
- (ii) The number of hits at a Web site in any time interval is a Poisson random variable. A particular site has on average $\lambda = 2$ hits per second. What is the probability that there are no hits in an interval of 0.25 seconds? What is the probability that there are no more than two hits in an interval of one second? (8)

Or

- (b) (i) The input voltage to a rectifier is a random variable U with a uniform distribution on $[-1,1]$. The rectifier output is a random variable W defined by $W = g(U) = \begin{cases} 0, & U < 0 \\ U, & U \geq 0 \end{cases}$. Find the cumulative distribution function of $F_w(w)$ and the expected value $E[W]$. (8)
- (ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modelled by an $N(5;16)$ normal random variable, X . What is the probability that a randomly selected parcel weighs between 1 and 10 pounds? What is the probability that a randomly selected parcel weighs more than 9 pounds? (8)
12. (a) The joint probability density function of the random variables X and Y is defined as follows: $f_{XY(x,y)} = \begin{cases} 25e^{-5y}, & 0 < x < 0.2, y > 0 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal density function of X and Y . Find the covariance of X and Y .

Or

- (b) Associated with a job are two random variables: CPU time required (Y) and the number of disk I/O operations (X). Given the following data, compute the sample correlation coefficient:

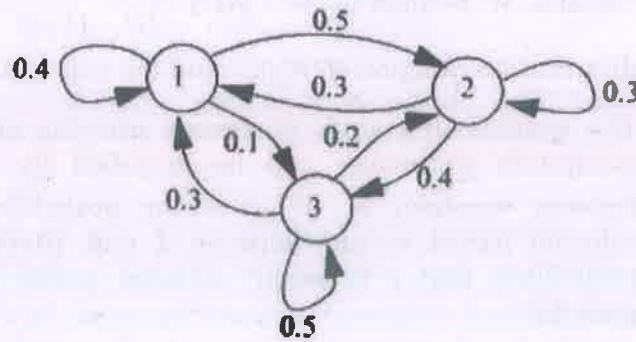
Number (X)	398	390	410	502	590	305	210	252	398	392
Time (Y)	40	38	42	50	60	30	20	25	40	39

Does a linear fit seem reasonable? Assuming that we wish to predict the CPU time requirement given an I/O request count, perform a linear regression: $Y = a + bX$.

13. (a) (i) At the receiver of an AM radio, the received signal contains a cosine carrier signal at the carrier frequency f_c with a random phase θ that is a sample value of the uniform $(0, 2\pi)$ random variable. The received carrier signal is $X(t) = A \cos(2\pi f_c t + \theta)$. What are the expected value and autocorrelation of the process $X(t)$? (8)
- (ii) X_n is a wide sense stationary random sequence with autocorrelation function $R_X[k]$. The random sequence Y_n is obtained from X_n by reversing the sign of every other random variable in X_n : $Y_n = -1^n X_n$. Express the autocorrelation function of Y_n in terms of $R_X[k]$ and express the cross-correlation function of X_n and Y_n in term of $R_X[k]$. Is Y_n wide sense stationary? (8)

Or

- (b) Find the transition probability matrix and the limiting-state probabilities of the process represented by the state-transition diagram shown in Figure.



14. (a) Let V denote the set of ordered pairs of real numbers. If (a_1, b_1) and (a_2, b_2) are elements of V and $c \in \mathbb{R}$, define $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 b_2)$ and $c(a_1, b_1) = (ca_1, b_1)$. Determine whether V is a vector space over $\mathbb{R}$ with these operations. If not, list the axioms that fail to hold.

Or

- (b) (i) Obtain the solution space, basis of the solution space, and its dimension of the system given below

$$A = \begin{pmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{pmatrix}. \quad (10)$$

- (ii) Show that the three vectors $u = (0, 3, 1, -1)$, $v = (6, 0, 5, 1)$ and $w = (4, -7, 1, 3)$ form a linearly dependent set in $\mathbb{R}^4$. (6)

15. (a) Assume that the vector space $\mathbb{R}^3$ has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors $u_1(1, -1, 1)$, $u_2 = (1, 0, 1)$, $u_3 = (1, 1, 2)$ into an orthogonal basis $\{v_1, v_2, v_3\}$ and then normalize the orthogonal basis vectors to obtain an orthonormal basis u_1, u_2 and u_3 .

Or

- (b) Find the least squares solutions, the least squares error vector and the least squares error of the linear system

$$3x_1 + 2x_2 - x_3 = 2,$$

$$x_1 - 4x_2 + 3x_3 = -2,$$

$$x_1 + 10x_2 - 7x_3 = 1$$